

AN ANALYTICAL STUDY OF ISLAMIC GEOMETRICAL PATTERNS
TOWARDS APPROPRIATE APPLICATION

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This work is dedicated to my beloved parents who have given me the opportunities and support throughout my life. It is also dedicated to my wonderful wife, who has always stood by me and dealt all our life difficulties with a smile

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ABSTRACT

Although Islam gives function and not form, but Islam as a context has effects on forms and ornaments in some ways. The great role of geometry in Islamic architecture due to restriction of using natural figures is an example. In this research, the application of Islamic geometrical patterns (IGPs), and suitability of their usage over architectural elements is studied. For this purpose, evolution of IGPs through history has been studied, while regional diversities are also taken into account. The research not only identifies origin of patterns but also reveals radical artistic movements through history of Islamic architecture. Therefore, suitability of patterns in terms of time-scale accuracy and period-style has been identified. The next step is focused on building elements and areas where patterns are applied. A survey of decorative patterns of hundred well-survived buildings through Muslim world of architecture has been conducted for this purpose, which reveals the types of patterns that are mostly used over different building elements. Then, the geometrical properties of the patterns are studied to find their similarities and differences. The results show, although IGPs are seems to be infinite types, however they are limited in terms of geometrical properties and can be classified. Therefore, the most prominent and widely used patterns collected have been analyzed further, based on their constructive polygons and polygon-tilings. It eventually simplified their overall view, which had significant help for their classification. It also helps to understand direction and nature of expansion of each pattern. The result also indicates that each pattern has a particular perfect outline proportion. For example, for a given dimension, not all, but particular patterns can be selected to fit and fulfill the area in a perfect proportion and without deformity along edges of a given area. Finally, this research managed to find guidelines to choose and fit IGPs into certain given areas over finishing surfaces of building elements.

ABSTRAK

Walaupun Islam memberikan fungsi dan bukan bentuk, tetapi dalam konteks Islam ada serba sedikit kesannya pada bentuk dan perhiasan. Peranan geometri dalam senibina Islam kerana sekatan menggunakan rupa semulajadi adalah satu contoh sebabnya. Dalam kajian ini, aplikasi corak geometri Islam (IGPs), dan kesesuaian penggunaan mereka ke atas unsur-unsur senibina telah dikaji. Bagi tujuan ini, evolusi IGPs melalui sejarah telah dikaji, manakala kepelbagaian serantau juga diambil kira. Kajian ini bukan sahaja mengenal pasti asal corak tetapi juga mendedahkan pergerakan seni radikal melalui sejarah senibina Islam. Oleh itu, kesesuaian corak dari segi kesesuaian masa dan tempoh gaya telah dikenal pasti. Langkah seterusnya memberi tumpuan kepada elemen bangunan dan kawasan di mana corak digunakan. Satu kajian terperinci terhadap seratus bangunan yang masih wujud dalam dunia senibina Islam telah dijalankan untuk tujuan ini, yang mendedahkan jenis corak yang kerap digunakan pada elemen-elemen bangunan yang berbeza. Selanjutnya, sifat geometri corak dikaji untuk mencari persamaan dan perbezaan mereka. Hasil kajian menunjukkan, walaupun IGPs seolah-olah menjadi jenis yang infiniti, namun mereka ada had dari segi sifat geometri dan ianya boleh diklasifikasikan. Oleh itu, corak yang paling menonjol dan digunakan secara meluas telah dikumpul dan dianalisa dengan lebih lanjut, berdasarkan poligon dan tiling-poligon yang membina mereka. Akhirnya pandangan corak mereka dipermudahkan secara keseluruhan, yang sangat membantu bagi mengklasifikasi corak ini. Ia juga membantu untuk memahami arah dan sifat pengembangan setiap corak. Hasil kajian juga menunjukkan bahawa setiap corak mempunyai perkadaran garis luar yang sempurna. Sebagai contoh, bagi dimensi yang diberikan, bukan semua, tetapi setengah corak tertentu boleh dipilih untuk disesuaikan dan memenuhi kawasan bahagian dalam dengan sempurna dan tanpa kecacatan disepanjang tepi kawasan yang diberikan. Akhirnya, kajian ini berjaya untuk mencari satu garis panduan untuk memilih dan memuatkan IGPs ke permukaan yang diberikan sebagai kemasan permukaan elemen bangunan dengan tepat.

TABLE OF CONTENT

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	xii
	LIST OF FIGURES	xiii
	LIST OF ABBREVIATIONS	xxvii
	LIST OF APPENDICES	xxviii
1	INTRODUCTION TO ISLAMIC GEOMETRICAL PATTERNS	1
	1.1. When & how did Geometry Infiltrate Into Islamic Architecture?	1
	1.2. Background	2
	1.3. Where Geometrical Patterns Have Been Used?	3
	1.4. Background of Research Problem	4
	1.5. Research Problem	6
	1.6. Research Objectives	7
	1.7. Research Methodology	8
	1.7.1. Data Collection	9
	1.7.2. Analysis	11
	1.8. Importance of Study	13

	1.9. Organization of Research	13
2	EVOLUTION OF IGPS	14
	2.1. Umayyad Architecture (660 – 750 CE)	16
	2.1.1. Great Architectural Works Survived	16
	2.2. Abbasids Architecture (750 – 1285 CE)	18
	2.2.1. Great Architectural Work Survived	19
	2.3. Seljuk Architecture (1083 – 1194 CE)	22
	2.3.1. Great Architectural Work Survived	23
	2.4. Fatimid’s Architecture (909 – 1171 CE)	25
	2.4.1. Great Architectural Work Survived	25
	2.5. Mamluks Architecture (1250 – 1517 CE)	27
	2.5.1. Great Architectural Work Survived	27
	2.6. Ottoman Architecture (1290 – 1923 CE)	31
	2.6.1. Great Architectural Work Survived	31
	2.7. Safavid Architecture (1501 – 1722 CE)	34
	2.7.1. Great Architectural Work Survived	35
	2.8. Mughal Architecture (1526 – 1737 CE)	39
	2.8.1. Great Architectural Work Survived	39
	2.9. Muslims of Spain	43
	2.10. Summary	44
3	CATEGORIZATION OF PATTERNS OVER FIVE MAJOR BUILDING ELEMENTS	46
	3.1. Dome	47
	3.2. Iwan, Portals & Openings	52
	3.3. Mihrab	57
	3.4. Minbar	63
	3.5. Minaret	70
	3.6. Summary	73
4	GEOMETRICAL ANALYSIS OF IGPS	75
	4.1. Symmetry Operations	76

4.1.1.	Reflection	76
4.1.2.	Rotation	77
4.1.3.	Translation	77
4.1.4.	Glide Reflection	78
4.1.5.	Scale Symmetry	78
4.2.	Wallpaper Patterning	80
4.2.1.	The Crystallographic Notation	81
4.2.2.	Symmetrical Groups (Based on IUC)	81
4.3.	Constructible and Non-constructible Patterns	86
4.4.	Polygon Tiling	87
4.5.	Grids in Islamic Geometrical Patterns	89
4.6.	Summary	92
5	ADVANCED CLASSIFICATION OF IGPS	93
5.1.	Collection of the Most Prominent Patterns	94
5.2.	Four & Eight Point Patterns	94
5.2.1.	Development of Eight-Point Pattern	96
5.2.2.	Eight-Point Square Filled	97
5.2.3.	Eight-Point Clover-Filled	98
5.2.4.	Eight-Point Star Cross-Filled	99
5.2.5.	Eight-Fold Rosette	101
5.2.6.	Eight-Point Star Clover-Filled	103
5.2.7.	Eight-Point Star Square-Filled	104
5.3.	Three & Six Point Patterns	105
5.3.1.	Development of Six-Point Pattern	106
5.3.2.	Regular Hexagon	107
5.3.3.	Six-Point Star (Trihexagonal Pattern)	108
5.3.4.	Six-Point Star Hexagon-Filled	108
5.3.5.	Six-Point Star Rhomb-Filled	109
5.3.6.	Rhombitrihexagonal Pattern	110
5.3.7.	Tripetalous Six-Point Star	111
5.4.	Five & Ten Point Patterns	112
5.4.1.	Pentagons in Tiling	114
5.4.2.	Cairo Tiling	114

5.5.	Ten-Point Pattern	115
5.5.1.	Development of Ten-Point Pattern	116
5.5.2.	Ten-Point Rosette Pattern	117
5.5.3.	Ten-Point Star Pattern	118
5.5.4.	Decagons and Pentagons Tiling	119
5.5.5.	D-P Tiling Type-1	120
5.5.6.	D-P Tiling Type-2	121
5.5.7.	D-P Tiling Type-3	123
5.5.8.	D-P Tiling Type-4	124
5.6.	Linking Elements in IGPs	126
5.7.	Design Variation	128
5.7.1.	Rendering	129
5.8.	Naming IGPs	130
5.9.	Summary	131

6	GUIDELINE TOWARDS MINIMIZING DEFORMITY AND PERFECT FRAME PROPORTION	133
6.1.	Unit-Cell	133
6.2.	Four Focal Centers Connecting Method	135
6.3.	Perfect Proportions of Five/Ten Point Patterns	136
6.3.1.	Ten-Point Star Pattern	136
6.3.2.	Ten-Point Rosette Pattern	139
6.3.3.	D-P Tiling Type-4	141
6.3.4.	D-P Tiling Type-1	142
6.3.5.	D-P Tiling Type-3	143
6.3.6.	D-P Tiling Type-4	144
6.3.7.	Perfect Proportions of Three/Six Point Patterns	145
6.3.8.	“Trihexagonal Six-Point Star” Pattern	145
6.3.9.	“Rhombitrihexagonal Six-Point Star” Pattern	146
6.3.10.	Tripetalous Six-Point Star	146
6.4.	Perfect Proportions of Four/Eight Point Patterns	147
6.4.1.	“Eight-Fold Rosette” Pattern	147
6.4.2.	“Four-Leaf Eight-Tie” Pattern	148
6.5.	Closing Remarks	149

7	CONCLUSIONS	151
	7.1. Evolution of IGPS and Time-Scale Accuracy	152
	7.2. Origin of IGPs and Architectural Style-Matching	152
	7.3. Categorization of IGPs Based on Five Major Building Elements	153
	7.4. Geometrical Properties, Similarities and Differences of IGPs	153
	7.5. Proper Application in Terms of Practical Issues	154
	7.6. Recommendations	155
	REFERENCES	157
	APPENDIX A	163

LIST OF TABLES

TABLE NO.	TITLE	PAGE
1.1	List of countries and number of buildings that has been selected from Books and literature reviews	10
1.2	Muslim Dynasties and Empires, which this study is focused on	10
1.3	Analysis Workflow diagram of research process	12
2.1	Time chart of evolution of IGPs through the history	45
3.1	Application of IGPs over Islamic building elements	74
4.1	Classification of patterns according to their lattice type	80
4.2	Group of symmetries, which generate unit-cell and the whole pattern, Dotted area, indicates the Primitive-cell of unit-cell of each pattern.	84
5.1	Classification of Islamic geometrical pattern	132
6.1	Proportion of IGPs based on “four focal centers connecting” method	149

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Demolishing Enghelab Square in Tehran, due to misunderstanding about historical roots of its geometrical motifs	5
1.2	Great mosque of Tehran, in Iran	5
1.3	Persada Johor (left), Sultan Hassan II mosque (center & right)	6
1.4	Hakim mosque of Isfahan in Iran 1662 CE	6
2.1	First level of Islamic geometrical pattern's classification (Source: Author)	15
2.2	Dome of Rock, Interior view of original decorations with floral motifs (Left) - New façade with geometrical design, date back to sixteen century (Right)	17
2.3	Great mosque of Damascus, 705 CE, aerial view & decorative facades mosaics	17
2.4	Al-Mshatta palace in Jordan, Decorative patterns of its southern entrance	18
2.5	The great mosque of Kairouan in Tunisia, 836 CE, Detail of carved floral patterns of original wooden Minbar	19
2.6	The great mosque of Kairouan in Tunisia, 836 CE, Decorative patterns of interior arcades (left), finishing surface of the original Mihrab (Right), shows the individual geometric shapes	20
2.7	Three Door Mosque in Kairouan, 866 CE, A view to the entrance and its decorative details	20

2.8	Ibn-Tulun Mosque in Egypt, 879 CE, The early example of eight point star geometrical pattern (First two from left) and six point geometry (third and forth from left)	21
2.9	Abbasid palace in Baghdad, 1230 CE, original geometric decorations before restoration	22
2.10	Tower of Kharaqan in Qazvin, 1093, 12-point, abstract 6 & 8-point geometrical patterns	23
2.11	Great Mosque of Isfahan in Iran, northeast dome and its interior design (Left & Center) - roof of praying hall dates back to twelfth century (Right)	24
2.12	Al-Juyushi Mosque in Egypt, 1085 CE, Spandrel of Mihrab (Left), interior decoration of dome (Right)	26
2.13	Advanced 6 and 8-point geometrical patterns over entrance door (Left) and ceiling (center) and the earliest example of Ten-point Star geometrical pattern in Mihrab of Qalawun complex in late 13th century	28
2.14	Mosque of Al-Nasir Mohammad in Cairo, 1334 CE, 10-point geometrical pattern used in decoration of Mihrab and window girls	29
2.15	Sultan Hassan complex in Cairo, 1363 CE, Sixteen-point geometrical patterns on the entrance doors	29
2.16	Sultan Qaybtay mosque in Cairo, 1475 CE, Carved wooden Minbar (Left, Center) - Carved geometrical pattern of the dome (Right)	30
2.17	Yesil Mosque in Bursa, 1421 CE, samples of 6, 8, and 10 point geometrical patterns	32
2.18	Bayezid Complex in Istanbul, 1508 CE, Ten & Nine-point Geometrical patterns	32
2.19	Shezade Complex in Istanbul, 1548 CE, showing different type of Ten-point geometrical pattern over wooden door and Minbar	33
2.20	Selimiye Complex in Edirne, 1575 CE, Few 6 & ten-point geometrical patterns over Minbar & window crowns	34

2.21	Ali-Qapu Palace in Isfahan, 1598 CE, Detail of balcony's ceiling	35
2.22	Sheikh Lutfallah Mosque in Isfahan, 1617 CE (Left-Center) - Shah Mosque in Isfahan, 1638 CE (Right)	36
2.23	Chehel-Sutun Palace in Isfahan, 1647 CE, fabulous 8 & 10-point geometrical patterns of the entrance ceiling	36
2.24	Hakim Mosque in Isfahan, 1662 CE, samples of 8-point geometrical patterns over the Iwans and vaults spandrels	37
2.25	Hakim Mosque in Isfahan, 1662 CE, samples of 10-point geometrical patterns over Northern Iwan (Left) and subsidiary Southern Iwan (Right)	37
2.26	Friday Mosque of Isfahan, 17 th century, Combination of 8 & 10-point geometrical pattern (Left), Combination of 8 & 12-point geometrical pattern (Right)	38
2.27	Friday Mosque of Isfahan, 17 th century, Samples of 10-point geometrical pattern through wall decorative tiles	38
2.28	Humayun Tomb in Delhi, Samples of six & 8-point geometrical patterns	40
2.29	Jahangiri-Mahal in Red Fort of Agra, 1570 CE, showing detailed 6 & 8-point geometrical patterns made by carved red sandstone	40
2.30	Red For in Agra, showing geometrical patterns used in its Amar-Singh Gate (First two from right) and lattice marble railing of Shish-Mahal (Right)	40
2.31	Friday mosque of Fatehpur-Sikri and Salim-Chishti tomb (inside the mosque), various 6,8,10 and rare 14-point star geometrical pattern	41
2.32	Tomb of Akbar-The Great in Sikandra, 1612 CE, geometric patterns of an Iwan of the tomb (Left) and the main gate (Right)	41
2.33	Etimad-ud-Daulah tomb in Agra, 1628 CE, celebration of 6 & 10 point geometrical patterns by inlay and carved marble through the facade of tomb	42
2.34	Lahore Fort in Pakistan, 17th century	42

2.35	Alhambra Palace in Spain, 1390 CE, showing details of 6, 8, 12 and 16-point geometrical patterns of tile and carved stucco decorative works	44
2.36	Alhambra Palace in Spain, 1390 CE, showing details of abstract concept of 6 and 8-point geometrical patterns	44
3.1	Isometric view of Shah Mosque in Isfahan, Iran, showing its most distinctive Islamic architectural elements	47
3.2	From left: interior apex of Friday mosque of Isfahan (12 th Century), Al Juyushi Mosque (1085 CE) in Egypt & Sultan Faraj-ibn-Barquq Complex (1411 CE) in Cairo	48
3.3	From Left: Mausoleums of Sultan al-Ashraf Barsbay, Amir Gani-Bak- al Ashrafi and Sultan Qaybtay complex	49
3.4	Sheikh Lutfallah Mosque in Isfahan, 1617 CE, showing the floral motifs of interior and exterior surfaces of its main dome	50
3.5	Friday mosque of Saveh (Left-Center) - Shah Nematollah Vali Shrine (Riht)	50
3.6	From Left: Friday Mosque of Yazd, Iran, 1375 CE –Hakim Mosque of Isfahan - Shir-Dar Madrasa in Uzbekistan, 1636 CE	51
3.7	Taj-Mahal mausoleum in India, 1653 CE (Left) – Badshahi mosque in Pakistan, 1671 CE (Right)	52
3.8	Entrance Iwan of Kalyan Mosque of Bukhara, 16th century (Left) – areas can be covered with decorative patterns over Iwans and openings (Right)	53
3.9	Arched openings of Abbasid palace (Left, Center) and Madrasa of Mustansiriya (Right)	53
3.10	Towers of Kharaqan in Qazvin, 1093 CE, showing the application of 6 & 12-point geometrical patterns inside blinded arched niches	54
3.11	Great Mosque of Isfahan in Iran, application of 8 & 10-point geometrical patterns covered top edges of its arched openings (Iwans)	54

- 3.12 Hakim Mosque in Isfahan, an 8-point over spandrels of southern Iwan (Left) and 10-point over spandrels of northern Iwan (Right) 56
- 3.13 From Left: Jahangiri-Mahal in Agra, Taj-Mahal mausoleum, Lahore Fort in Pakistan 56
- 3.14 From left: Great Mosque of Cordoba, 785-987 CE - Aljaferia Palace, 11th century - Alhambra Palace in Spain, 1390 CE 57
- 3.15 Great mosque of Damascus, 705 CE, The Mihrab of Companion; application of six, eight and 12-point geometrical patters 58
- 3.16 Great mosque of Damascus, 705 CE, The main Mihrab; application of 6, 8 and 12-point geometrical patters 58
- 3.17 Friday Mosque of Ardestan in Iran, 11th century - Great Mosque of Isfahan in Iran, 12th century - Friday Mosque of Ashtarjan in Iran, 1316 CE 59
- 3.18 Friday Mosque of Barsian in Iran, Mihrab is decorated by rarest Islamic geometrical patterns 59
- 3.19 From left: Al-Azhar Mosque, Zaytuna Mosque, Al-Juyushi Mosque, Ibn-Tulun Mosque 60
- 3.20 From Left; Mihrabs of Qalawun mosque, Al-Nasir Mohammad Mosque, Amir Qijmas Al-Ishaqi Mosque & Sultan Qansuh al-Ghuri Complex 61
- 3.21 From left: Rustam Pasha Mosque in Istanbul, 1563 CE - Selimiye Complex in Edirne, 1575 CE - Mihrab of Yesil Mosque in Bursa 61
- 3.22 Friday Mosque of Yazd, 1365 CE (Left) - Sheikh Lutfallah Mosque in Isfahan, 1617 CE (Center) - Hakim Mosque in Isfahan, 1662 CE (Right) 62
- 3.23 From Left: Delhi Friday Mosque (1644-58 CE) - Moti Mosque in Lahore, 1645 CE - Taj-Mahal mausoleum in Agra (1653 CE) - Badshahi Mosque in Lahore, 1674 CE 62
- 3.24 Fatehpur-Sikri mosque (Left) - Tilla Kari Madrasa (Center) - Mosque of Khiva (Right) 63

- 3.25 Al-Aqsa Mosque (Left) - Great mosque of Damascus (Center), Ibn-Tulun Mosque (Right) 64
- 3.26 From left: Alaeddin Mosque in Konya, Divrigi Mosque & Nain Friday mosque 65
- 3.27 Friday Mosque of Isfahan, Left Minbar dates to 16th century during Safavids ear and right side Minbar dates to 14th century. 65
- 3.28 Marble Minbars of Shah Mosque Isfahan 17th century (Left) & Vakil Mosque in Shiraz 18th century (Right) 66
- 3.29 From left: Mosque of Al-Nasir Mohammad, Minbar of Sultan Hassan complex & Amir Qijmas Al-Ishaqi Mosque, all in Cairo, 67
- 3.30 Wooden Minbars of Al-Muayyad (Left & Center) & Sultan Qaybtay mosque (Right) 67
- 3.31 From left: Ulu-Cami of Bursa, Bayezid II Mosque in Istanbul, Sehzade Mosque in Istanbul 68
- 3.32 Minbars of Selimiye Complex in Edirne (Left) & Mihrimah Sultan Mosque (Right) 69
- 3.33 From left: Jamat Khana Mosque, Friday Mosque of Delhi, and Friday Mosque of Irich 69
- 3.34 From left: Damghan Friday Mosque in Iran, 1080 CE - Chihil Dukhtaran Minaret in Isfahan, Iran 1108 CE - Saveh Friday mosque in Iran, 1110 CE - Sarban Minaret in Isfahan, Iran 1155 CE - Ali Mosque in Isfahan, Iran 12th century - Jam minaret (1195 CE) in Afghanistan 70
- 3.35 From Left: Shah Mosque in Isfahan, 1638 CE - The Great Mosque of Isfahan, 17th century – “Madar e Shah” Madrasa in Isfahan, 1714 CE - Al-Azhar Mosque - Bab-al-zuwayla in Cairo, 1092 CE - Al-Nasir Mohammad in Cairo, 1334 CE - Qaybtay Complex 71
- 3.36 From Left: Shezade mosque in Istanbul, 1548 CE – Suleymaniye complex in Istanbul, 1558 CE – Qutb Minar in Delhi – Taj-Mahal in

	Agra, 1653 CE – Friday Mosque of Delhi, 1658 CE – Badshahi mosque in Lahore, 1674 CE – Hiran Minar – Itimad-ud-Daulah tomb	72
4.1	Four basic symmetry operations	76
4.2	Reflection symmetry in Islamic geometrical patterns (Left), Rotational symmetry in tiles of Friday Mosque of Isfahan, Iran (Right)	77
4.3	Alhambra, in Spain, 1391 CE, Translational symmetry in walls patterns	78
4.4	Ibn-Tulun Mosque in Egypt, 879 CE (Left) – Alhambra Palace, in Spain, 1391 CE (Center), Friday Mosque of Isfahan, Iran, The use of Scale variation technique in ceiling decoration (Right)	79
4.5	Axes of Translation (Top left), Axes of Reflection (Top right), Axes of Glide Symmetry (bottom left), Points of Rotational Symmetry (bottom right)	79
4.6	Five possible regular tiling; (From Field and Golubitsky, 2009)	80
4.7	From Left: Decagon, 10-point star, 10-point star with 2 layers leaves, 10-fold Rosette	87
4.8	Patterns created with square, triangle and hexagon have no gap or spaces in between (in other words Regular tilings)	87
4.9	Uniform or Semi-regular tiling (Figures by R.A. Nonenmacher, Wikipedia eng.)	88
4.10	In this example the circle grid does not fit the long rectangular unit of pattern. Figure From El-Said (1993), Geometric Concepts in Islamic Art”	90
4.11	Pattern over Khaja Abdullah Ansari shrine in Afghanistan, 1429 CE (Left) - Complete set of Girih tiles proposed by Peter J. Lu (A: decagon, B: pentagon, C: rhombus, D: hexagon, E: bowtie) (Right)	91
4.12	Topkapi scroll, Late 15th or early 16th century. where Peter Lu has pointed out and colored Girih tiles	91
5.1	Steps to create an Eight-point Star	95

5.2	Eight-point star framing Simurgh (Bird) Cinili Kiosk Museum, Turkey	95
5.3	A popular type of Eight-Point Star geometrical patter, showing it has intertwined constructive circular system at the background.	96
5.4	Eight-point square filled, octagons spreading diagonally.	97
5.5	Eight-point square field, the gaps fill with squared tile or elements with sides equal to octagon sides (left), Imam-Zamin Tomb in Delhi, India, 16th century, showing carved window grille with Eight-point square field design (Right).	97
5.6	Steps to construct Eight-Point Clover filled. Source: Author	98
5.7	Eight-Point Clover filled Step 4 (Left), Eight-Point Clover, filled by square and triangles (Center), Eight-Point Clover filled (Right)	99
5.8	Mosque-Madrassa of Sultan Hassan in Cairo. Source: Author	99
5.9	Showing Eight-point cross-field with constructive interlacing circles and lines, it is one of the most popular type of eight-point geometrical Patterns	100
5.10	Eight-Point star cross-filled, gaps can be filled either by cross element (Right) or combination of a squares and triangles or even irregular pentagons (Left)	100
5.11	Shir-Dar Madrasa in Samarkand, Uzbekistan, 17th century (Left) - 13th century tile work in Kashan, Iran (Right). Source: Author	101
5.12	Eight-fold rosette. Source: Author	101
5.13	Square-filled eight fold pattern filled by eight-fold rosette (Left), extending the lines of rosette vertices to shape regular octagons (Center), Eight-fold Rosette pattern (Right). Source: Author	102
5.14	Qal'a Madrasa in Baghdad (Left) - Arslanhane mosque in Sivas (Right)	102
5.15	Extending sides of Eight-point star to make secondary layer of leaves (Left), Eight-point Star Clover-filled (Right) Source: Author	103

5.16	Pool of Diwan-i-Khas in Lahore fort, Pakistan, 1645 CE (Left) - Maghak-i-Attari Mosque in Bukhara, Uzbekistan, 12th century (Right)	103
5.17	Eight-point plumb star pattern. Source: Author	104
5.18	Tilla-Kari Madrasa in Samarkand.	104
5.19	Steps to set-up regular hexagon with compass and straightedge	105
5.20	Six-point star with its constructive circle grids (Left) - Qutaish mosque in Sidon, Lebanon, 15 th century, showing Six-point star in a ceramic tile (Right)	105
5.21	Development of six-point geometrical pattern using interlacing circles (left), The hexagon can form a regular tessellate, having 3 hexagons around every vertex and no gap in between (right). Source: Author	106
5.22	Types of uniform and hexagon tilings	107
5.23	Lahore Fort, 17 th century, shows Marble screen of regular hexagon	107
5.24	Dome of Rock in (Left) - Qutaish Mosque (Right)	108
5.25	Consider a Trihexagonal tiling; remove the hexagon boundary of some hexagons in a way that six hexagons surround a six-point star.	109
5.26	Lahore Fort (Left) - The Alai Darwaza (Right)	109
5.27	From left: Six-point star rhomb-filled polygon tiling, final pattern, Darwish Pasha Mosque in Damascus, Detail of 17 th century Iranian Painted miniature. Source: Author	109
5.28	Rhombitrihexagonal tiling (Left) is constructive base of a popular Islamic Six-point geometry (Right)	110
5.29	Six-point geometry based on Rhombitrihexagonal tiling (Left) – Tilla- Kari Madrasa and Mosque in Samarkand, Uzbekistan, 17th century CE (Right)	110
5.30	Tripetalous Six-point Star	111

5.31	Tripetalous Six-point Star (Top left) - Karatay Madrasa in Konya, Turkey, 13th century (Top right) - A mosaic tile of Lahore Fort in Pakistan (Bottom)	111
5.32	The ratio of Minor and major chords of the pentagon circumscribed by a circle, is $\phi = 1:1.618$	112
5.33	Constructing a regular pentagon	113
5.34	The Tomb of Akbar in Agra	114
5.35	Constructing a regular pentagon (Left), & a regular decagon (Right)	115
5.36	Various decagon tilings	116
5.37	Placing ten-point rosette inside decagons that are connected to each other along their vertices, in this pattern decagons are connected to each other over their vertices and spread diagonally. Source: Author	117
5.38	Another pattern developed by decagons which are connected to each other over their vertices in zigzag manner (Left) - Mosaic decor from Gök Madrasa in Sivas, Turkey, 13th century CE (Right). Source: Author	118
5.39	Placing a ten-point star (double layer of leaves) in a pattern created by connecting decagons over their vertices (Left), Extending the sides of star to shape the linking elements (Center), Ten-point star pattern showing the 2 layers of leaves and linking elements (Right). Source: Author	119
5.40	Mausoleum of Itimad-al-Daula in Agra, 17th century CE (Left) - Ceramic façade of Mausoleum of Zangi Ata in Tashkent, 14th century CE (Right)	119
5.41	A popular type of tiling consisted of pentagon and decagons	120
5.42	Pattern developed by decagons and pentagons with ratio of 1:6	120
5.43	Placing “10-point star” over decagon-pentagon tiling (Left) - Extending sides of stars (Right). Source: Author	121

5.44	Ten-point star pattern based on decagon and pentagon tiling (Left), Details of wooden ceiling Chihil Sutun, Palace in Isfahan, Iran 17 th Century CE (Right). Source: Author	121
5.45	Pattern developed by decagons and pentagons with ratio of 1:8	122
5.46	Placing “10-point star” over decagon-pentagon tiling (Left), extending sides of star to form pattern (Right)	122
5.47	Ten-Point Star pattern Based on D-P Tiling Type-2 (Left) - Mahrough Tomp in Nishapur, Iran, 17 th century CE showing tile decoration (Right)	122
5.48	Placing “10-point rosette” over decagon-pentagon tiling (Left), extending sides of rosettes to form “long 10-point Rosette” pattern (Right)	123
5.49	“Long Ten-point Rosette” pattern (Left) - Chihil Sutun, Palace in Isfahan, Iran 17 th Century CE, showing the detailing of wooden ceiling (Right)	123
5.50	“Long Ten-point rosette” (left) Vs regular Ten-point rosette (Right)	124
5.51	Unit-cell of a Decagon-pentagon tiling (Left), Development of D-P tiling (Right)	125
5.52	Decagon-Pentagon tiling that acts as constructive base for this pattern	125
5.53	“Deformed Drum Pattern” Is a type of 10-point star pattern (Left) – Agha Nur mosque in Isfahan, Iran, 17 th century (Right)	125
5.54	The most prominent linking element in Islamic geometrical pattern	127
5.55	“Four-Phoebus & Kohl Container” pattern, setup process and linking elements. Source: Author	128
5.56	Dome of Friday mosque of Saveh, Iran	129
5.57	Various Rendering Styles (picture from Kaplan, 2000)	129
5.58	From Grid Method Classification of Islamic Geometric Patterns Naming method invented by Ahmad M. Aljamali and Ebad Banissi (2003)	130

- 6.1 Primitive-cell, Unit-cell and full pattern. Source: Author 134
- 6.2 “Obtuse ten-point star”, four adjacent stars and their centers (Left) - Connecting center of stars where the ratio of long to short chard (i.e. b/a) is 1:1.3764 which is equal to its unit-cell sides ratio (Right). Source: Author 135
- 6.3 Ten-point star pattern showing the 2 layers of leaves and linking elements (Left) - Mausoleum of Itimad-al-Daula in India, 17th century (Right). Source: Author 136
- 6.4 Components of “obtuse ten-point star” and their popular portions for using along boundaries (1.618 : Golden Ratio). Source: Author 137
- 6.5 Extracted Unit-cell of pattern (Left) - “Obtuse ten-point star” (Right) 138
- 6.6 Kites (light blue) are cut in half and ten-point star (dark blue) and drum (yellow) has been cut in quarter along the edges of outline boundaries. 138
- 6.7 “Ten-point Rosette”, showing the four adjacent rosette and their centers (Left) - Connecting center of rosettes, the ratio of long to short chard (i.e. b/a) is 1:1.3764 which is equal to its unit-cell sides ratio, Right. Source: Author 139
- 6.8 Unit-cell with ratio of 1:1.3764 (Left) “Ten-point Rosette” pattern (Right) 139
- 6.9 The figure shows that how both “Obtuse ten-point star” (Top) & “Ten-point Rosette” pattern, (bottom) are based on octagon tiling of the same arrangement. 140
- 6.10 “Deformed Drum” pattern, showing four adjacent stars and their centers (Left) Connecting center of stars where the ratio of short to long chard (Right) 141
- 6.11 Unit-cell with ratio of 1:1.3764 (Left) - “Deformed Drum” pattern, Left (Right) - Ten-point star is cut in quarter, kite, pentagon and Container have been cut in half along the edges of outline boundaries (Bottom, Left). Source: Author 141

- 6.12 “Four-Phoebus” pattern, four adjacent stars and their centers (Left) - Connecting center of stars where the ratio of long to short chord (i.e. b/a) is 1:1.1756 which is equal to its unit-cell sides ratio, Right. Source: Author 142
- 6.13 Unit-cell with ratio of 1:1.1756 - “Four-Phoebus” pattern (Right) 142
- 6.14 “Long Four-Phoebus” pattern, showing the four adjacent stars and their centers (Left) - Connecting center of stars where the ratio of long to short chord (i.e. b/a) is 1:2.0262 which is equal to its unit-cell sides ratio (Right). Source: Author 143
- 6.15 Unit-cell (Left) - “Long Four-Phoebus” pattern (Right). Source: Author 143
- 6.16 “Long ten-point Rosette” pattern, four adjacent stars and their centers (Left) - Connecting center of stars where the ratio of long to short chord (i.e. b/a) is 1:2.0262 which is equal to its unit-cell sides ratio (Right). Source: Author 144
- 6.17 Unit-cell with ratio of 1:2.0262 (Left) - Long Ten-point Rosette (Right) 144
- 6.18 From left: Unit-cell, “Trihexagonal six-point star” pattern, showing the four adjacent stars and their centers, Connecting center of stars. Source: Author 145
- 6.19 From Left: Unit-cell of “ Rhombitrihexagonal Six-point star”, four adjacent stars and their centers, Connecting center of stars where the ratio of long to short chord (i.e. $b/a = d/c$) is 1:1.7321 which is equal to its unit-cell sides ratio 146
- 6.20 From Left: Unit-cell of “Tripetalous Six-point Star”, Pattern with four adjacent stars and their centers, Connecting center of stars where the ratio of long to short chord (i.e. $b/a = d/c$) is 1:1.7321 which is equal to its unit-cell sides ratio 146
- 6.21 Unit-cell with ratio of 1:1 (Left), “Eight-fold rosette” pattern (Right) 147
- 6.22 “Eight-fold rosette” pattern, four adjacent stars and their centers (Left) - Connecting center of stars where the ratio of long to short chord (i.e.

	b/a is 1:1 which is equal to its unit-cell sides ratio (Right). Source: Author	147
6.23	Unit-cell with ratio of 1:1 (Left), “Four-leaf Eight-tie” pattern (Right)	148
6.24	“Four-leaf Eight-tie” pattern, four adjacent stars and their centers (Left) - Connecting center of stars where the ratio of long to short chord (i.e. b/a) is 1:1 which is equal to its unit-cell sides ratio (Right). Source: Author	148
7.1	Application of golden proportions in design of Sahn dimensions	156

LIST OF ABBREVIATIONS

CE	Common Era
D-P Tiling	Decagon Pentagon Tiling
IGPs	Islamic Geometrical Patterns
IUC	International Union Of Crystallography

LIST OF APPENDICES

TABLE NO.	TITLE	PAGE
A	List of Analyzed Buildings	164

CHAPTER 1

INTRODUCTION TO ISLAMIC GEOMETRICAL PATTERNS

1.1. When & How Did Geometry Infiltrate Into Islamic Architecture?

"For without symmetry and proportion no temple can have a regular plan," Ancient roman architect, Marcus Vitruvius Pollio (80 BC to 15 BC) wrote in his famous treatise *De Architectura*. *"Twenty years were spent in erecting the pyramid itself: of this, which is square, each face is eight plethora (is a measurement used in Ancient times, equal to 100 Greek feet), and the height is the same; it is composed of polished stones, and jointed with the greatest exactness; none of the stones are less than thirty feet."* – Father of history **Heroditus** (484 BC-425 BC) belived to be first who wrote about Pyramids which mostly built around 2800 B.C. (Calter, 2008). Upon those mentioned examples, we can state that integration of geometry and architecture has been existed and understood long before the birth of Islam. Nevertheless, we cannot dismiss the important of geometry through Muslims' history of architecture. But now we can reform a new statement: Why is geometry so important in Islamic Architecture? In addition, why such particular geometries are called Islamic and when they become popular in world of Islamic architecture?

The expansion and development of geometry through Islamic art and architecture can be related to significant growth of the scientific and technological innovations of eight and ninth centuries in Middle East, Iran and central Asia by translations of ancient texts from languages such as Greek and Sanskrit into Arabic

(Turner, 1997 Page 27). By the tenth century, original Muslim contributions to the sciences became significant; in this context, important developments in the field of geometry resulted from the work of, among others, Umar-al-Khayyam, Abu'l Wafa al-Buzjani, Abu Mansur al-Khwarizmi and Ibn-al-Haytham (Özdural, 1995; Mohamed, 2000). Although our knowledge of the development of the science of geometry in the pre-modern Islamic world is considerable, we do not know enough about the processes and intermediaries through which theoretical geometric knowledge was transferred to an applied field such as architecture.

It is believed that the earliest written documents on geometry through the Islamic history of science, is Khwarizmi's mathematical book; *Al-Kitāb al-mukhtaṣar fī hīsāb al-ğabr wa'l-muqābala* (The Compendious Book on Calculation by Completion and Balancing, written in early ninth century) (Mohamed, M., 2000, page 17-43). Hence it is not surprising to see a gap of nearly three centuries from the rise of Islam in the early seventh century (Berkey, 2003, Page 3) to late ninth century which earliest example of decorative geometrical patterns can be traced over survived buildings of Muslim world (See "Time chart of evolution of IGPs through the history" in Table 2.1).

1.2. Background

The background of Islamic architecture is a continues motion of architecture, starts form eastern movement of Greek and Roman architecture merged with local styles of Achaemenid, Partians, Sassanians (Fletcher and Cruickshank, 1996). In early seventh century in Arabian province of Roman Empire, the explosive expansion of Islam introduced the new religion throughout northeast into Mesopotamia, Persia and Asia Minor, west to Egypt and Mediterranean Sea and along the coast of Africa. Regions conquered By Muslims were already developed their own construction techniques and were rich in knowledge of adapting natural resources for constructional purposes (Frishman, 1994). It shows that, although geometric decorations have been developed significantly by Muslims, the basic

geometrical knowledge and constructive shapes had already existed from Byzantine and Sassanian empires (Met, 2004). Through this research, the influence and efforts of Muslim architects, craftsmen and patrons of major Muslim Empire & dynasties towards creation and development of Islamic Building decorations and particularly geometrical patterns have been studied. Some minor dynasties (in terms of their impact) such as Buyid, Ayubids, Ilkhanid, Timurid etc, are neglected and would be studied in detail in forward researches.

1.3. Where Geometrical Patterns Have Been Used?

Before starting study of applications of geometrical patterns in Islamic architecture, we have to specify its elements. Elements of Islamic architecture can be classified into two main categories of “building” and “decorative & ornamental” elements. Through the history of Muslim architecture, there are some common elements in buildings, which can be pointed out. Iwan, Arches, Riwaq, Domes, Sahn, lighting, fountains etc. are some of building parts that can be found in both religious and secular architecture of Muslim world. There are some other elements such as Minarets, Minbar, Mihrab, Maqsura etc. but they are particularly mosques and madras’s elements (Grube and Michell, 1995).

Although Islamic architecture has experienced variety of taste, styles of regions from Indian subcontinent to African west coast during its history, some distinguishable characteristics have preserved which is called as Islamic architectural style. Among those traits, the principle role of surface decoration is undoubtedly the most magnificent character (Clévenot and Degeorge, 2000).

Islamic decorative patterns can be classified as figural and non-figural types, which the non-figural patterns include ornamental **geometric**, **calligraphic** and **floral** patterns. Another type of decorative element is Muqarnas that is closely linked to that of two-dimensional geometric patterns. In third chapter, application of different decorative patterns over elements of Islamic architecture, with

concentration of geometrical patterns is studied. However, it is good to notice that all Islamic architectural elements can be analyzed from either two-dimensional view in terms of their architectural drawings or from three-dimensional view in terms of their relation and proportions to other existing elements of that building or space.

1.4. Background of Research Problem

Geometrical patterns are one of the key ornamental elements in Islamic architecture. They are widely used as decorative elements over building elements in the Islamic world of architecture. There is no doubt that the glory, beauty and balance that Islamic architecture is famous for, is based on intellectual interplay of mathematical and geometrical sciences, with art and architecture. However, the question of suitability and appropriate use of these patterns in terms of both philosophical and practical issues remained ambiguous and unanswered. The issue becomes more crucial when we observe that through the modern world of Islamic architecture, geometrical patterns are copied all over the surface of architectural elements and the only changes that can be noticed are new materials and constructional methods which help architects to scale patterns and cover larger spaces. Yet it is not clear that using a particular pattern for decorating domes, floors, walls, screens etc. is suitable in terms of period-style, regional-style or even common in terms of spaces, location and elements which they are going to use over? In addition, what we observe in contemporary Islamic architecture as “copy-pasting” patterns all over any surfaces to create a spiritual ambiance is a wise choice?

An example of misunderstanding of historical roots and origins of Islamic geometrical patterns (IGPs) was the decorative pattern of Enghelab (Revolution) Square in Tehran, Iran. The six-point geometrical pattern, which has been used in that square, is one of the earliest types of IGPs and can be found in the Ibn-Tulun mosque in Cairo which belongs to the late ninth century. However, lack of knowledge about the history of IGPs and their evolution, made a great misunderstanding among Iranian Media

and authorities, that ended up to demolishing that square. Figure 1.1 shows that how municipal authorities responded to medias criticism.



Figure 1.1 Demolishing Enghelab Square in Tehran, due to misunderstanding about historical roots of its geometrical motifs

In terms of timescale accuracy, periods' style and property, Great Mosque of Tehran is an interesting example. Its design is inspired from distinctive Seljuk architecture (1037-1194 CE). However, decorative patterns designed for decorating exterior surfaces of its main courtyard is a type ten-point geometrical pattern (Figure 1.2), which were not common during Seljuks and was not in favor of Seljuk architects and artisans.

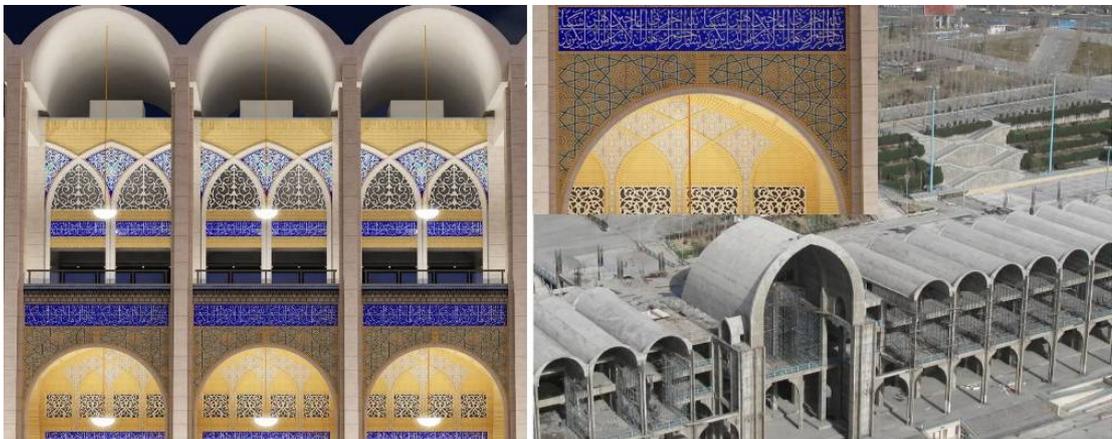


Figure 1.2 Great mosque of Tehran, in Iran

Apart from philosophical issues, IGPs are often used inappropriately in terms of scale, dimension and fitting inside frames. Often patterns are scaled and roughly fitted without concerning broken and deformed shapes along edges of surfaces that result lack of perfection. Figure 1.3 shows how patterns are applied imperfectly in Persada Johor in Malaysia and Sultan Hassan II mosque in Morocco while stars and

polygons are cut roughly and remained casual. Comparing such cases with elegant survived architectural masterpieces such as Hakim mosque of Isfahan (1656-62 CE) (Figure 1.4) shows that using patterns is far beyond only pasting over a random surface. Architects used to first analyze geometry of surfaces, then patterns were modified and adopted specifically in the way that fulfill the surface perfectly and with minimum deformity along the edges. In other words, stars and shapes are cut logically (either half or quarter) with designers order and not accidentally. Above examples shows necessity of studies toward establishing guidelines on use and suitability Islamic geometrical patterns in Islamic inspired buildings.



Figure 1.3 Persada Johor (left), Sultan Hassan II mosque (center & right)



Figure 1.4 Hakim mosque of Isfahan in Iran 1662 CE

1.5. Research Problem

For centuries, Islamic geometrical patterns are used as decorative elements over walls, ceilings, grilles, doors and openings, dome, minarets, etc. However,

having no guideline and code for these adorable ornaments, caused often-inappropriate use, in terms of time-scale accuracy, period-style, scale, dimensioning and even identity. This research would like to investigate IGPs both historically and mathematically to answer questions regarding suitability and appropriate use of these patterns as buildings decorative elements. In this regard, questions, which this research tried to answer, are:

- When were IGPs introduced to Islamic architecture? In addition, when does each different type of IGPs introduced to Muslim architects and artisans?
- Where did patterns develop and by whom?
- Which building elements are more decorated and by what pattern? I.e. where they have been used?
- What are geometrical properties of IGPs? What are their characteristic, Similarities and Differences?
- How should we use IGPs? And how we can reduce practical defects?

1.6. Research Objectives

This research aims to establish a guideline of appropriate use of Islamic geometrical patterns in Islamic inspired buildings. To be close to our goal following objectives must be achieved;

1. First Objective is to sketch Evolution of IGP patterns through history of Islamic Architecture.

2. Second Objective is to study influence of regionalism and Major Muslim Dynasties to Identify Origin of Different types of patterns
3. Another important objective is to categories patterns according their medium i.e. building elements, which they are mostly, applied over.
4. One of the most challenging Objectives of this study is to find geometrical properties of IGPs and classify them practically and with applied architectural language
5. Final objective was to find a method to minimize deformity of patterns while applying over required surfaces

1.7. Research Methodology

This research is based on descriptive approaches for which our goal was to gather survived geometrical patterns (data), and classify them to find how these architectural heritages have been threated through history of Islamic architecture and how should we used them in contemporary Islamic inspired buildings. To establish a guideline to application of IGPs, the first step is to find their origin in terms of both period and regional style. This would result dialectic answers to wide ranges of philosophical and architectural questions, such as what period? or where? a particular pattern was so popular and has been widely used. For this purpose ornaments of 100 famous buildings (Data) has been collected and classified based on time-scale and regionalism to find popular patterns of different architectural eras and styles.

The next target was to find on which parts of building IGPs are mostly used. Hence, information regarding five main Islamic architectural elements and how they have been treated in those 100 buildings gathered, classified based on types of elements and analyzed. To find geometrical properties of IGPs, geometrical issues related to decorative patterns have been pointed out and studied to find their

application in IGPs. Results have been further analyzed to find a suitable way to classify patterns and establish a method of appropriate use in practical terms.

1.7.1. Data Collection

To archive above-mentioned objective, a detailed survey decorative patterns of well-survived monument (either religious or secular) form all around the world of Islamic architecture must be conducted. On literature review bases a collection of 100 well-known and survived buildings from West Africa coast to Indian subcontinent, over historical span nearly twelve century from early stages of Islam to late 18th century has been selected. This wide ranged collection cover most important classic architectural treasures of Islamic world. For this purpose not only encyclopedias of history of architectures (such as: *A History of Architecture* by Sir Banister Fletcher, 1996 ; *Architecture of Islamic World* by George Michell, 1995 ; *Color and Symbolism In Islamic Architecture* by Roland Michaud, 1996 ; *Islamic Art and Architecture From Isfahan to Taj-Mahal* by Henri Stierlin, 2002 ; *Monuments of Civilization* by Umberto Scerrato, 1977 ; *Ornament and decoration in islamic architecture* by Dominique Clevenot, 2000 and etc.) in one hand, but regional/local architectural studies (such as: *A History of Ottoman Architecture* by John Freely, 2011 ; *Architecture of Mughal India* by Catherine B. Asher, 1992 ; *Islamic Architecture in Cairo* by Doris Behrens-Abouseif, 2007; and etc.) has been deeply investigated to extract 100 splendid architectural treasures which can represent glory of Islamic architecture within scope of this research. Selected countries and buildings are statistically demonstrated in Table 1.1.

Moreover, above, buildings are going to be studied based on influence of dynasties and empires, which buildings' patrons were, belonged. This would outfit this research, by answer of favored patterns of each Muslim Empires and dynasties and even a great help to find reasons behind wide popularity of some patterns. Table 1.2 shows major dynasties that are studied through following chapters of this research.

Table 1.1 List of countries and number of buildings that has been selected from Books and literature reviews

	Countries	No. of Buildings
1	Afghanistan	1
2	Egypt	21
3	India	11
4	Iran	25
5	Iraq	3
6	Jordan	2
7	Pakistan	4
8	Palestine	2
9	Spain	4
10	Syria	3
11	Tunisia	3
12	Turkey	16
13	Turkmenistan	1
14	Uzbekistan	4
	Total	100

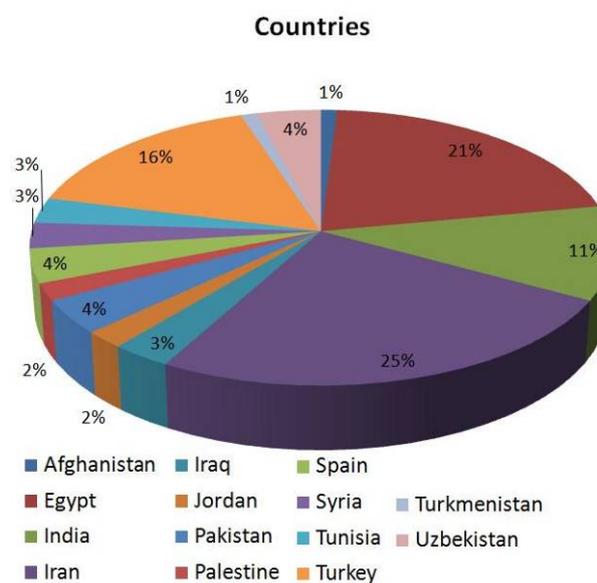
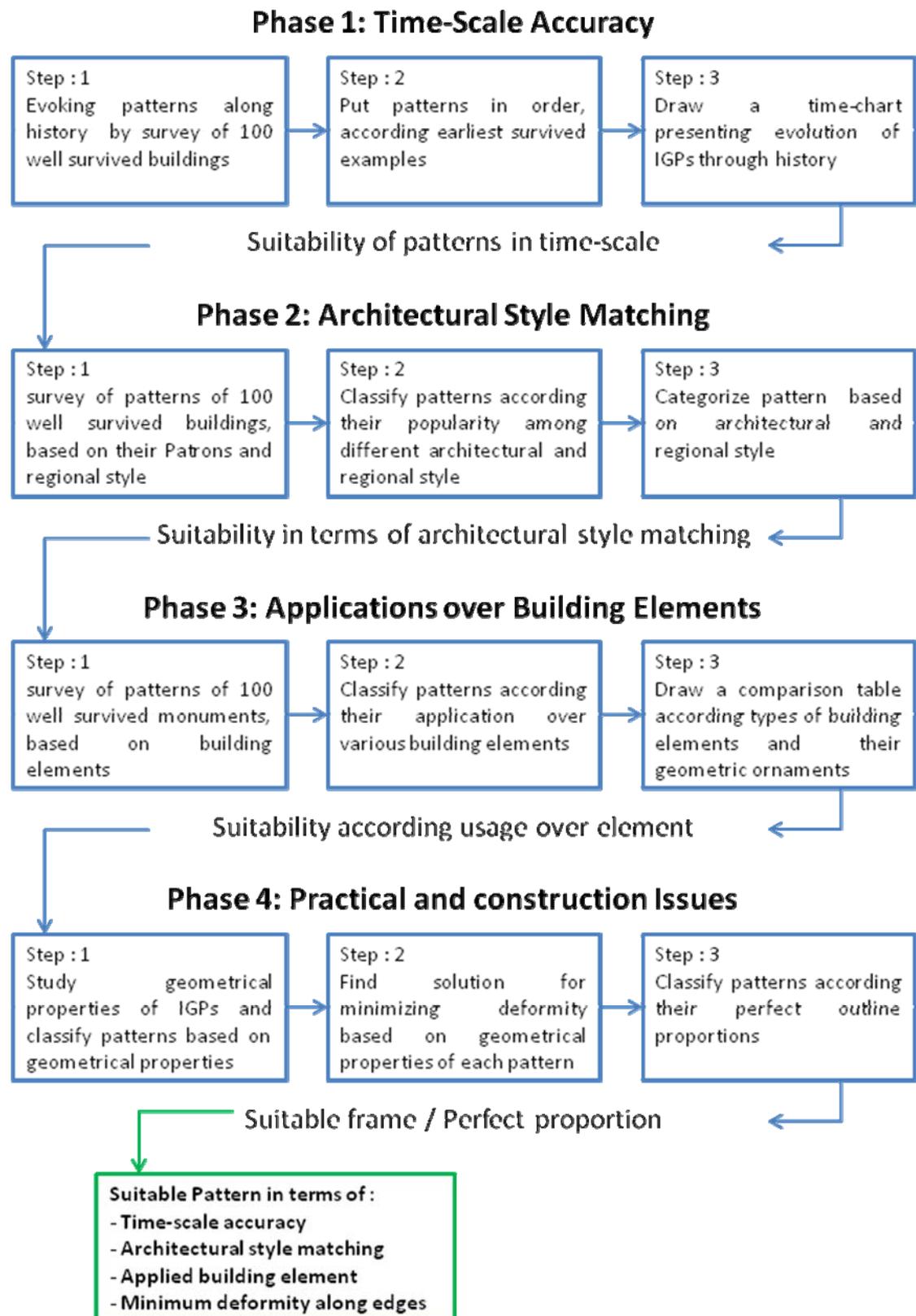


Table 1.2 Muslim Dynasties and Empires, which this study is focused on

	Style/Period	Capitals	Established	Disestablished
1	Umayyads	Damascus, Córdoba	660	750
2	Abbasid	Baghdad	750	1258
4	Fatimid	Mahdia, Cairo	909	1171
5	Seljuks	Ray, Isfahan, Hamadan, Kerman	1038	1194
6	Mamluk	Cairo	1250	1517
7	Ottoman	Söğüt, Bursa, Edirne, Constantinople	1290	1923
8	Safavid	Tabriz, Qazvin, Esfahan	1501	1736
9	Mughal	Agra; Fatehpur Sikri; Delhi	1526	1858

1.7.2. Analysis

Subsequently, application of patterns over above-mentioned buildings observed, recorded and documented according their date and region along with type of building elements (Such as dome, Minaret and etc.), which they are applied over. After collecting and documenting data, the **first analytical phase** is to tabulate patterns in terms of time-scale and put them in order. Eventually results of this phase lead us to draw a time-chart of evolution of patterns based on earliest survived examples. **Second analytical phase** was to discover and identify origin of patterns. In this stage, collected patterns are reclassified based on regionalism and where they are found. The result of this classification allows us to categorize patterns based on architectural and regional styles. **Third analytical phase** of this research is to characterize patterns according to where these patterns are mostly used in buildings. This will establish a guideline to use of IGPs upon building elements and would answer to “which parts of buildings are more decorated with patterns and what types of patterns are common for such elements?”. For this purpose, five main Islamic architectural elements have been selected. Dome would represent methods and techniques of decoration three-dimensional elements; Iwan is selected to cover two-dimensional elements such as walls and arched openings and windows. Mihrab is great example for Islamic interior designs and Minbar is a sample of Islamic building furniture. In addition, Minaret is selected to demonstrate accessibility and visual issues of decorating elements by geometrical patterns. As a result, third chapter of this research is focused on classifying and categorizing patterns by building elements, which they have been applied. While types of patterns in terms of style-period and place-to-apply are identified, it comes to question of “how to apply and adopt patterns for required surfaces?”. Hence, at **fourth analytical phase**, geometrical properties of patterns are analyzed based on geometrical issues that are related to architecture and ornaments. A detailed literature review on geometrical issues of architecture and ornaments in general and specifically IGPs is carried out in forth chapter of this research and as a result patterns ore classified according to their geometrical properties in fifth chapter. Moreover, in sixth chapter a guideline and method to simple way of recognizing suitability of patterns based on geometrical properties is provided.

Table 1.3 Analysis Workflow diagram of research process

1.8. Importance of Study

This research is an answer to ambiguities regarding both philosophical and practical aspects of suitability and adapting Islamic geometrical patterns. It traces evolution of Islamic patterns from early stages nearly modern era and identifies the origin of popular patterns. This research has collected the most prominent types of IGPs and categorized them according to their usage over building elements. Geometrical properties of patterns are also analyzed, and based on results, patterns are classified according to their geometrical properties. Eventually a guideline has been established to verify suitability of patterns in terms of style, period, types of elements that they are applied over and finally similarities and differences between patterns. Moreover, a method to adopt these patterns to required area and a quick way of recognizing type of patterns in terms of geometrical properties is provided to help practical issues of using these types of ornaments.

1.9. Organization of Research

This report is organized into seven parts. Apart from first introductory and final conclusionary chapter, the five remaining chapters are sorted to address from the most fundamental issue of “when (evolution) and from where (origin)?” to last stage of “how to use?”. In Second chapter, IGPs are studied historically based on both time-scale and period-styles, to draw time-chart of evolution of Islamic geometrical patterns. Third chapter is focused on building elements and categorizing patterns according to where they are mostly used in Islamic inspired buildings. In fourth chapters, geometrical properties of patterns are studied, and as a result, in chapter five, patterns are classified according to their similarities and differences of geometrical properties. Throughout sixth chapter, a method for covering surfaces with minimum deformity along edges has been introduced. In addition, a simple way to find the perfect ratio for frame (outline boundary) of patterns is also introduced and illustrated with samples of patterns from different types of IGPs.

REFERENCES

Books:

- Abas, S. J. and A. S. Salman (1995). *Symmetries of Islamic geometrical patterns*. World Scientific.
- Akkach, S. (2005). *Cosmology and architecture in premodern Islam: an architectural reading of mystical ideas*. State University of New York Press.
- Alfieri, B. M. and F. Borromeo (2000). *Islamic architecture of the Indian Subcontinent*. Laurence King Pub.
- Antoniou, J. (1998). *Historic Cairo: a walk through the Islamic city*. The American University in Cairo Press.
- Asher, C. E. B. (1992). *Architecture of Mughal India*. Cambridge University Press.
- Behrens-Abouseif, D. (2007). *Cairo of the Mamluks: a history of the architecture and its culture*. I.B. Tauris.
- Berkey, J. P. (2003). *The formation of Islam: religion and society in the Near East, 600-1800*. Cambridge University Press.
- Blair, S. and J. M. Bloom (1995). *The art and architecture of Islam 1250-1800*. Yale University Press.
- Brend, B. (1991). *Islamic art*. Harvard University Press.
- Broug, E. (2008). *Islamic geometric patterns*. Thames & Hudson.
- Burton-Page, J. and G. Michell (2008). *Handbook of oriental studies: India. Indian Islamic architecture: forms and typologies, sites and monuments*. Brill.
- Byron, R. (1982). *The road to Oxiana*. Oxford University Press.
- Calter, P. A. (2008). *Squaring the Circle: Geometry in Art and Architecture*. John Wiley & Sons.

- Clévenot, D. and G. Degeorge (2000). *Ornament and decoration in Islamic architecture*. Thames & Hudson.
- Conway, J. H., H. Burgiel, et al. (2008). *The symmetries of things*. A.K. Peters.
- Cosman, M. P. and L. G. Jones (2008). *Handbook to life in the medieval world*. Facts On File.
- Critchlow, K. (1989). *Islamic Patterns (An Analytical and Cosmological Approach)*. London: Thames & Hudson Ltd.
- Dixon, R. (1987). *Mathographics*. Dover Publications.
- El-Said, I., T. El-Bouri, et al. (1993). *Islamic art and architecture: the system of geometric design*. Garnet Pub.
- Ertuğ, A., M. T. Bragner, et al. (1991). *The Seljuks: a journey through Anatolian architecture*. Ahmet Ertuğ.
- Ettinghausen, R., O. Grabar, et al. (2001). *Islamic art and architecture 650-1250*. Yale University Press.
- Ettinghausen, R., O. Grabar, et al. (2001). *Islamic art and architecture 650-1250*. Yale University Press.
- Field, M. and M. Golubitsky (2009). *Symmetry in chaos: a search for pattern in mathematics, art, and nature*. Society for Industrial and Applied Mathematics.
- Fletcher, B. and D. Cruickshank (1996). *Sir Banister Fletcher's a history of architecture*. Architectural Press.
- Flood, F. B. (2001). *The Great Mosque of Damascus: studies on the makings of an Umayyad visual culture*. Brill.
- Frederickson, G. N. (2003). *Dissections: plane & fancy*. Cambridge University Press.
- Freely, J. (2011). *History of Ottoman Architecture*. WIT Press.
- Frishman, M., H. U. Khan, et al. (1994). *The mosque: history, architectural development & regional diversity*. Thames and Hudson.
- Frishman, M., H. U. Khan, et al. (1994). *The mosque: history, architectural development & regional diversity*. Thames and Hudson.
- Gauss, C. F. and W. C. Waterhouse (1966). *Disquisitiones arithmeticae*. Springer-Verlag.

- Gibilisco, S. (2003). *Geometry Demystified*. McGraw-Hill.
- Golombek, L. and D. Wilber (1988). *The timurid architecture of Iran and Turan*. Princeton University Press.
- Goodwin, G. (1971). *A history of Ottoman architecture*. Johns Hopkins Press.
- Goodwin, G. (1991). *Islamic Spain*. Penguin.
- Grube, E. J. and G. Michell (1995). *Architecture of the Islamic world: its history and social meaning : with a complete survey of key monuments and 758 illustrations, 112 in colour*. Thames and Hudson.
- Grunbaum, B. and G. C. Shephard (2011). *Tilings and Patterns*. Dover Publications.
- Hill, D. and O. Grabar (1967). *Islamic architecture and its decoration A.D. 800-1500: a photographic survey*. Faber.
- Hillenbrand, R. (1999). *Islamic art and architecture*. Thames and Hudson.
- Jeffrey B. Spurr and Riedlmayer, B. a. J. S. (1994). *Resources for the Study of Islamic Architecture*. Cairo, The Aga Khan Program for Islamic Architecture.
- Krebs, R. E. (2004). *Groundbreaking scientific experiments, inventions, and discoveries of the Middle Ages and the Renaissance*. Greenwood Press.
- Kuban, D. (1974). *Iconography of religions: Islam. Muslim religious architecture. The mosque and its early development*. Brill.
- Kuban, D. and C. Emden (2010). *Ottoman architecture*, Antique Collectors' Club.
- Lapunzina, A. (2005). *Architecture of Spain*. Greenwood Press.
- Madden, T. F. (2002). *The Crusades: the essential readings*. Blackwell.
- Mainzer, K. (2005). *Symmetry and complexity: the spirit and beauty of nonlinear science*. World Scientific.
- Martin, G. E. (1998). *Geometric constructions*. Springer.
- Met, T. and Y. U. Press (2004). *Islamic Art and Geometric Design: Activities for Learning with Other and Booklet*. YALE UNIV PR.
- Michaud, R., M. Barry, et al. (1996). *Colour and symbolism in Islamic architecture: eight centuries of the tile-maker's art*. Thames and Hudson.
- Mohamed, M. (2000). *Great Muslim mathematicians*. Universiti Teknologi Malaysia.

- Petersen, A. (1996). *Dictionary of Islamic architecture*. Routledge.
- Scerrato, U. (1977). *Monuments of civilization: Islam*. Cassell.
- Sherbaf, A. (2006). *Gereh & Karbandi (Ties & Squinches' Design)*. Tehran: Sobhan Pub.
- Sinclair, T. A. (1990). *Eastern Turkey: an architectural and archaeological survey*. Pindar Press.
- Steinhaus, H. (1999). *Mathematical Snapshots*. Dover Publications.
- Stierlin, H. and A. Stierlin (2002). *Islamic art and architecture*. Thames & Hudson.
- Tabbaa, Y. (1997). *Constructions of power and piety in medieval Aleppo*. Pennsylvania State University Press.
- Turner, H. R. (1997). *Science in medieval Islam: an illustrated introduction*. University of Texas Press.
- Vermeulen, U., D. D. Smet, et al. (1995). *Egypt and Syria in the Fatimid, Ayyubid and Mamluk eras, Uitgeverij Peeters*. Behrens-Abouseif, D. *Islamic Architecture in Cairo: An Introduction*. Brill.
- Williams, C. (2008). *Islamic monuments in Cairo: the practical guide*. American University in Cairo Press.
- Williams, K. (2007). *Nexus Network Journal 9,1: Architecture and Mathematics*. Birkhäuser Verlag AG.
- Williams, R. (1979). *The geometrical foundation of natural structure: a source book of design*. Dover Publications.
- Yeomans, R. (2006). *The art and architecture of Islamic Cairo*. Garnet Pub. Ltd.
- Zee, A. and R. Penrose (2007). *Fearful symmetry: the search for beauty in modern physics*. Princeton University Press.

Articles & studies:

- Abas, S. J. (2001). Islamic geometrical patterns for the teaching of mathematics of symmetry. *Symmetry: Culture and Science*. Vol. 12, Pages 53-65.

- Ahmad M. Aljamali, Ebad Banissi (2003). *Grid Method Classification of Islamic Geometric Patterns*. South Bank University, London
- Baer, E. (1985). *The Mihrab in the Cave of the Dome of the Rock*. Muqarnas, Vol. 3, Pages 8-19. BRILL.
- Baer, E. (1999). *The human figure in early Islamic art: Some preliminary remarks*. Muqarnas. Vol. 16, Pages 32-41. BRILL.
- D'Avennes, P. and G. T. Scanlon (2008). *Islamic Art in Cairo: From the Seventh to the Eighteenth Centuries*. American University in Cairo Press.
- DeTemple, D. W. (1991). *Carlyle Circles and the Lemoine Simplicity of Polygon Constructions*. *The American Mathematical Monthly*. Vol. 98(2) , Pages 97-108.
- Edwards, C. and D. Edwards (1999). *The Evolution of the Shouldered Arch in Medieval Islamic Architecture*. *Architectural History* Vol. 42, Pages 68-95. SAHGB Publications Limited.
- Eric T. Eekhoff . *Constructability of Regular Polygons*. Iowa State University - MATH 599 - Creative Component.
- Grafman, R. and M. Rosen-Ayalon (1999). *The Two Great Syrian Umayyad Mosques: Jerusalem and Damascus*. Muqarnas Vol. 16, Pages 1-15. BRILL.
- Helmer Aslaksen. *Mathematics in Art and Architecture GEK1518*. Department of Mathematics, National University of Singapore
- Kaplan, C. S. (2000). *Computer Generated Islamic Star Patterns*. *Bridges 2000: Mathematical Connections in Art, Music and Science*.
- Kaplan, C. S. (2005). *Islamic star patterns from polygons in contact*. *Proceedings - Graphics Interface*. Pages 177-185.
- Kaplan, C. S. and D. H. Salesin (2004). *Islamic star patterns in absolute geometry*. *Acm Transactions on Graphics*. Vol. 23(2), Pages 97-119.
- Lu, P. J. and P. J. Steinhardt (2007). *Decagonal and quasi-crystalline tilings in medieval Islamic architecture*. *Science*. Vol. 315(5815), Pages 1106-1110.
- Ostromoukhov, V. (1998). *Mathematical tools for computer-generated ornamental patterns*. *Electronic Publishing, Artistic Imaging, and Digital Typography*. Springer Berlin / Heidelberg. Vol. 1375, Pages 193-223.
- Özdural, A. (1995). *Omar Khayyam, Mathematicians, and "Conversazioni" with Artisans*. *Journal of the Society of Architectural Historians*. Vol. 54(1), Pages 54-71.

- Peter J. Lu, Paul J. Steinhardt (2007). Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. *Science*, Vol. 315, Pages 1106-1110.
- Petitjean, M. (2007). A Definition Of Symmetry. *Symmetry: Culture and Science*. Vol. 18, Pages 99-119.
- Rabbat, N. (1996). Al-Azhar Mosque: An architectural chronicle of Cairo's history. *Muqarnas*. Vol. 13, Pages 45-67.
- Schattschneider, D. (1992). The Fascination Of Tiling. *Leonardo*. Vol. 25(3-4), Pages 341-348.
- Sugimoto, T. & T. Ogawa (2000). Tiling Problem of Convex Pentagon. *Forma*, Vol. 15, Pages 75–79.
- The Metropolitan Museum of Art (2004). *Islamic Art and Geometric Design; Activities for learning*. New York: The Metropolitan Museum of Art.
- WK. Chorbachi (1989). In the tower of Babel beyond symmetry in Islamic design. *Computers and Mathematics with Applications*, Vol. 17, Pages 751-789.