AN ANALYTICAL STUDY OF ISLAMIC GEOMETRICAL PATTERNS TOWARDS APPROPRIATE APPLICATION

YAHYA ABDULLAHI

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This work is dedicated to my beloved parents who have given me the opportunities and support throughout my life. It is also dedicated to my wonderful wife, who has always stood by me and dealt all our life difficulties with a smile

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ABSTRACT

Although Islam gives function and not form, but Islam as a context has effects on forms and ornaments in some ways. The great role of geometry in Islamic architecture due to restriction of using natural figures is an example. In this research, the application of Islamic geometrical patterns (IGPs), and suitability of their usage over architectural elements is studied. For this purpose, evolution of IGPs through history has been studied, while regional diversities are also taken into account. The research not only identifies origin of patterns but also reveals radical artistic movements through history of Islamic architecture. Therefore, suitability of patterns in terms of time-scale accuracy and period-style has been identified. The next step is focused on building elements and areas where patterns are applied. A survey of decorative patterns of hundred well-survived buildings through Muslim world of architecture has been conducted for this purpose, which reveals the types of patterns that are mostly used over different building elements. Then, the geometrical properties of the patterns are studied to find their similarities and differences. The results show, although IGPs are seems to be infinite types, however they are limited in terms of geometrical properties and can be classified. Therefore, the most prominent and widely used patterns collected have been analyzed further, based on their constructive polygons and polygon-tilings. It eventually simplified their overall view, which had significant help for their classification. It also helps to understand direction and nature of expansion of each pattern. The result also indicates that each pattern has a particular perfect outline proportion. For example, for a given dimension, not all, but particular patterns can be selected to fit and fulfill the area in a perfect proportion and without deformity along edges of a given area. Finally, this research managed to find guidelines to choose and fit IGPs into certain given areas over finishing surfaces of building elements.

ABSTRAK

Walaupun Islam memberikan fungsi dan bukan bentuk, tetapi dalam konteks Islam ada serba sedikit kesannya pada bentuk dan perhiasan. Peranan geometri dalam senibina Islam kerana sekatan menggunakan rupa semulajadi adalah satu contoh sebabnya. Dalam kajian ini, aplikasi corak geometri Islam (IGPs), dan kesesuaian penggunaan mereka ke atas unsur-unsur senibina telah dikaji. Bagi tujuan ini, evolusi IGPs melalui sejarah telah dikaji, manakala kepelbagaian serantau juga diambil kira. Kajian ini bukan sahaja mengenal pasti asal corak tetapi juga mendedahkan pergerakan seni radikal melalui sejarah senibina Islam. Oleh itu, kesesuaian corak dari segi kesesuaian masa dan tempoh gaya telah dikenal pasti. Langkah seterusnya memberi tumpuan kepada elemen bangunan dan kawasan di mana corak digunakan. Satu kajian terperinci terhadap seratus bangunan yang masih wujud dalam dunia senibina Islam telah dijalankan untuk tujuan ini, yang mendedahkan jenis corak yang kerap digunakan pada elemen-elemen bangunan yang berbeza. Selanjutnya, sifat geometri corak dikaji untuk mencari persamaan dan perbezaan mereka. Hasil kajian menunjukkan, walaupun IGPs seolah-olah menjadi jenis yang infiniti, namun mereka ada had dari segi sifat geometri dan ianya boleh diklasifikasikan. Oleh itu, corak yang paling menonjol dan digunakan secara meluas telah dikumpul dan dianalisa dengan lebih lanjut, berdasarkan poligon dan tiling-poligon yang membina mereka. Akhirnya pandangan corak mereka dipermudahkan secara keseluruhan, yang sangat membantu bagi mengklasifikasi corak ini. Ia juga membantu untuk memahami arah dan sifat pengembangan setiap corak. Hasil kajian juga menunjukkan bahawa setiap corak mempunyai perkadaran garis luar yang sempurna. Sebagai contoh, bagi dimensi yang diberikan, bukan semua, tetapi setengah corak tertentu boleh dipilih untuk disesuaikan dan memenuhi kawasan bahagian dalam dengan sempurna dan tanpa kecacatan disepanjang tepi kawasan yang diberikan. Akhirnya, kajian ini berjaya untuk mencari satu garis panduan untuk memilih dan memuatkan IGPs ke permukaan yang diberikan sebagai kemasan permukaan elemen bangunan dengan tepat.

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CE	Common Era
D-P Tiling	Decagon Pentagon Tiling
IGPs	Islamic Geometrical Patterns
IUC	International Union Of Crystallography

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CHAPTER 1

INTRODUCTION TO ISLAMIC GEOMETRICAL PATTERNS

1.1. When & How Did Geometry Infiltrate Into Islamic Architecture?

"*For without symmetry and proportion no temple can have a regular plan*," Ancient roman architect, Marcus Vitruvius Pollio (80 BC to 15 BC) wrote in his famous treatise *De Architectura*.*"Twenty years were spent in erecting the pyramid itself: of this, which is square, each face is eight plethora (*is a measurement used in Ancient times, equal to 100 Greek feet)*, and the height is the same; it is composed of polished stones, and jointed with the greatest exactness; none of the stones are less than thirty feet."* – Father of history **Heroditus** (484 BC-425 BC) belived to be first who wrote about Pyramids which mostly built around 2800 B.C. (Calter, 2008). Upon those mentioned examples, we can state that integration of geometry and architecture has been existed and understood long before the birth of Islam. Nevertheless, we cannot dismiss the important of geometry through Muslims' history of architecture. But now we can reform a new statement: Why is geometry so important in Islamic Architecture? In addition, why such particular geometries are called Islamic and when they become popular in world of Islamic architecture?

The expansion and development of geometry through Islamic art and architecture can be related to significant growth of the scientific and technological innovations of eight and ninth centuries in Middle East, Iran and central Asia by translations of ancient texts from languages such as Greek and Sanskrit into Arabic

(Turner, 1997 Page 27). By the tenth century, original Muslim contributions to the sciences became significant; in this context, important developments in the field of geometry resulted from the work of, among others, Umar-al-Khayyam, Abu'l Wafa al-Buzjani, Abu Mansur al-Khwarizmi and Ibn-al-Haytham (Özdural, 1995; Mohamed, 2000). Although our knowledge of the development of the science of geometry in the pre-modern Islamic world is considerable, we do not know enough about the processes and intermediaries through which theoretical geometric knowledge was transferred to an applied field such as architecture.

It is believed that the earliest written documents on geometry through the Islamic history of science, is Khwarizmi’s mathematical book; Al-Kitāb al-mukhtaṣar fī hīsāb al-ğabr wa’l-muqābala (The Compendious Book on Calculation by Completion and Balancing, written in early ninth century) (Mohamed, M., 2000, page 17-43). Hence it is not surprising to see a gap of nearly three centuries from the rise of Islam in the early seventh century (Berkey, 2003, Page 3) to late ninth century which earliest example of decorative geometrical patterns can be traced over survived buildings of Muslim world (See “Time chart of evolution of IGPs through the history” in Table 2.1).

1.2. Background

The background of Islamic architecture is a continues motion of architecture, starts form eastern movement of Greek and Roman architecture merged with local styles of Achaemenid, Partians, Sassanians (Fletcher and Cruickshank, 1996). In early seventh century in Arabian province of Roman Empire, the explosive expansion of Islam introduced the new religion throughout northeast into Mesopotamia, Persia and Asia Minor, west to Egypt and Mediterranean Sea and along the coast of Africa. Regions conquered By Muslims were already developed their own construction techniques and were rich in knowledge of adapting natural resources for constructional purposes (Frishman, 1994). It shows that, although geometric decorations have been developed significantly by Muslims, the basic geometrical knowledge and constructive shapes had already existed from Byzantine and Sassanian empires (Met, 2004). Through this research, the influence and efforts of Muslim architects, craftsmen and patrons of major Muslim Empire & dynasties towards creation and development of Islamic Building decorations and particularly geometrical patterns have been studied. Some minor dynasties (in terms of their impact) such as Buyid, Ayubids, Ilkhanid, Timurid etc, are neglected and would be studied in detail in forward researches.

1.3. Where Geometrical Patterns Have Been Used?

Before starting study of applications of geometrical patterns in Islamic architecture, we have to specify its elements. Elements of Islamic architecture can be classified into two main categories of "building" and "decorative & ornamental" elements. Through the history of Muslim architecture, there are some common elements in buildings, which can be pointed out. Iwan, Arches, Riwaq, Domes, Sahn, lighting, fountains etc. are some of building parts that can be found in both religious and secular architecture of Muslim world. There are some other elements such as Minarets, Minbar, Mihrab, Maqsura etc. but they are particularly mosques and madras's elements (Grube and Michell, 1995).

Although Islamic architecture has experienced variety of taste, styles of regions from Indian subcontinent to African west coast during its history, some distinguishable characteristics have preserved which is called as Islamic architectural style. Among those traits, the principle role of surface decoration is undoubtedly the most magnificent character (Clévenot and Degeorge, 2000).

Islamic decorative patterns can be classified as figural and non-figural types, which the non-figural patterns include ornamental **geometric**, **calligraphic** and **floral** patterns. Another type of decorative element is Muqarnas that is closely linked to that of two-dimensional geometric patterns. In third chapter, application of different decorative patterns over elements of Islamic architecture, with concentration of geometrical patterns is studied. However, it is good to notice that all Islamic architectural elements can be analyzed from either two-dimensional view in terms of their architectural drawings or form three-dimensional view in terms of their relation and proportions to other existing elements of that building or space.

1.4. Background of Research Problem

Geometrical patterns are one of key ornamental element is Islamic architecture. They are widely used as decorative elements over building elements in Islamic world of architecture. There is no doubt that the glory, beauty and balance that Islamic architecture is famous for, is based on intellectual interplay of mathematical and geometrical sciences, with art and architecture. However, the question of suitability and appropriate use of these patterns in terms of both philosophical and practical issuers remained ambiguous and unanswered. The issue becomes more crucial when we observe that through modern world of Islamic architecture, geometrical patterns are copied all over the surface of architectural elements and the only changes that can be noticed are new materials and constructional methods which help architects to scale patterns and cover larger spaces. Yet it is not clear that using a particular pattern for decorating domes, floors, walls, screens etc. is suitable in terms of period-style, regional-style or even common in terms of spaces, location and elements which they are going to use over? In addition, what we observe in contemporary Islamic architecture as “copy-pasting” patterns all over any surfaces to create a spiritual ambiance is a wise choice?

An example of misunderstanding of historical roots and origins of Islamic geometrical patterns (IGPs) was decorative pattern of Enghelab (Revolution) Square in Tehran, Iran. The six-point geometrical pattern, which has been used in that square, is one of the earliest type of IGPs and can be found in Ibn-Tulun mosque in Cairo which belongs to late ninth century. However, lack of knowledge about history of IGPs and their evolution, made a great misunderstanding among Iranians Media and authorities, that ended up to demolishing that square. Figure 1.1 shows that how municipal authorities responded to medias criticism.

Figure 1.1 Demolishing Enghelab Square in Tehran, due to misunderstanding about historical roots of its geometrical motifs

In terms of timescale accuracy, periods' style and property, Great Mosque of Tehran is an interesting example. Its design is inspired from distinctive Seljuk architecture (1037-1194 CE). However, decorative patterns designed for decorating exterior surfaces of its main courtyard is a type ten-point geometrical pattern (Figure 1.2), which were not common during Seljuks and was not in favor of Seljuk architects and artisans.

Figure 1.2 Great mosque of Tehran, in Iran

Apart from philosophical issues, IGPs are often used inappropriately in terms of scale, dimension and fitting inside frames. Often patterns are scaled and roughly fitted without concerning broken and deformed shapes along edges of surfaces that result lack of perfection. Figure 1.3 shows how patterns are applied imperfectly in Persada Johor in Malaysia and Sultan Hassan II mosque in Morocco while stars and polygons are cut roughly and remained casual. Comparing such cases with elegant survived architectural masterpieces such as Hakim mosque of Isfahan (1656-62 CE) (Figure 1.4) shows that using patterns is far beyond only pasting over a random surface. Architects used to first analyze geometry of surfaces, then patterns were modified and adopted specifically in the way that fulfill the surface perfectly and with minimum deformity along the edges. In other words, stars and shapes are cut logically (either half or quarter) with designers order and not accidently. Above examples shows necessity of studies toward establishing guidelines on use and suitability Islamic geometrical patterns in Islamic inspired buildings.

Figure 1.3 Persada Johor (left), Sultan Hassan II mosque (center & right)

Figure 1.4 Hakim mosque of Isfahan in Iran 1662 CE

1.5. Research Problem

For centuries, Islamic geometrical patterns are used as decorative elements over walls, ceilings, grilles, doors and openings, dome, minarets, etc. However, having no guideline and code for these adorable ornaments, caused often-inappropriate use, in terms of time-scale accuracy, period-style, scale, dimensioning and even identity. This research would like to investigate IGPs both historically and mathematically to answer questions regarding suitability and appropriate use of these patterns as buildings decorative elements. In this regard, questions, which this research tried to answer, are:

- When were IGPs introduced to Islamic architecture? In addition, when does each different type of IGPs introduced to Muslim architects and artisans?

- Where did patterns develop and by whom?

- Which building elements are more decorated and by what pattern? I.e. where they have been used?

- What are geometrical properties of IGPs? What are their characteristic, Similarities and Differences?

- How should we use IGPs? And how we can reduce practical defects?

1.6. Research Objectives

This research aims to establish a guideline of appropriate use of Islamic geometrical patterns in Islamic inspired buildings. To be close to our goal following objectives must be achieved;

1. First Objective is to sketch Evolution of IGP patterns through history of Islamic Architecture.

2. Second Objective is to study influence of regionalism and Major Muslim Dynasties to Identify Origin of Different types of patterns

3. Another important objective is to categories patterns according their medium i.e. building elements, which they are mostly, applied over.

4. One of the most challenging Objectives of this study is to find geometrical properties of IGPs and classify them practically and with applied architectural language

5. Final objective was to find a method to minimize deformity of patterns while applying over required surfaces

1.7. Research Methodology

This research is based on descriptive approaches for which our goal was to gather survived geometrical patterns (data), and classify them to find how these architectural heritages have been threated through history of Islamic architecture and how should we used them in contemporary Islamic inspired buildings. To establish a guideline to application of IGPs, the first step is to find their origin in terms of both period and regional style. This would result dialectic answers to wide ranges of philosophical and architectural questions, such as what period? or where? a particular pattern was so popular and has been widely used. For this purpose ornaments of 100 famous buildings (Data) has been collected and classified based on time-scale and regionalism to find popular patterns of different architectural eras and styles.

The next target was to find on which parts of building IGPs are mostly used. Hence, information regarding five main Islamic architectural elements and how they have been treated in those 100 buildings gathered, classified based on types of elements and analyzed. To find geometrical properties of IGPs, geometrical issues related to decorative patterns have been pointed out and studied to find their application in IGPs. Results have been further analyzed to find a suitable way to classify patterns and establish a method of appropriate use in practical terms.

1.7.1. Data Collection

To archive above-mentioned objective, a detailed survey decorative patterns of well-survived monument (either religious or secular) form all around the world of Islamic architecture must be conducted. On literature review bases a collection of 100 well-known and survived buildings from West Africa coast to Indian subcontinent, over historical span nearly twelve century from early stages of Islam to late 18th century has been selected. This wide ranged collection cover most important classic architectural treasures of Islamic world. For this purpose not only encyclopedias of history of architectures (such as: A History of Architecture by Sir Banister Fletcher, 1996 ; Architecture of Islamic World by George Michell, 1995 ; Color and Symbolism In Islamic Architecture by Roland Michaud, 1996 ; Islamic Art and Architecture From Isfahan to Taj-Mahal by Henri Stierlin, 2002 ; Monuments of Civilization by Umberto Scerrato, 1977 ; Ornament and decoration in islamic architecture by Dominique Clevenot, 2000 and etc.) in one hand, but regional/local architectural studies (such as: A History of Ottoman Architecture by John Freely, 2011 ; Architecture of Mughal India by Catherine B. Asher, 1992 ; Islamic Architecture in Cairo by Doris Behrens-Abouseif, 2007; and etc.) has been deeply investigated to extract 100 splendid architectural treasures which can represent glory of Islamic architecture within scope of this research. Selected countries and buildings are statistically demonstrated in Table 1.1.

Moreover, above, buildings are going to be studied based on influence of dynasties and empires, which buildings' patrons were, belonged. This would outfit this research, by answer of favored patterns of each Muslim Empires and dynasties and even a great help to find reasons behind wide popularity of some patterns. Table 1.2 shows major dynasties that are studied through following chapters of this research.

Table 1.1 List of countries and number of buildings that has been selected from Books and literature reviews

	Countries	No. of Buildings
1	Afghanistan	1
2	Egypt	21
3	India	11
4	Iran	25
5	Iraq	3
6	Jordan	2
7	Pakistan	4
8	Palestine	2
9	Spain	4
10	Syria	3
11	Tunisia	3
12	Turkey	16
13	Turkmenistan	1
14	Uzbekistan	4
Total		**100**

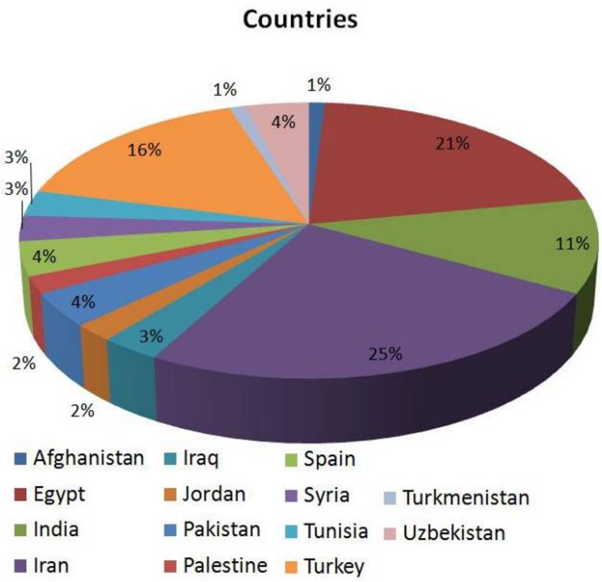


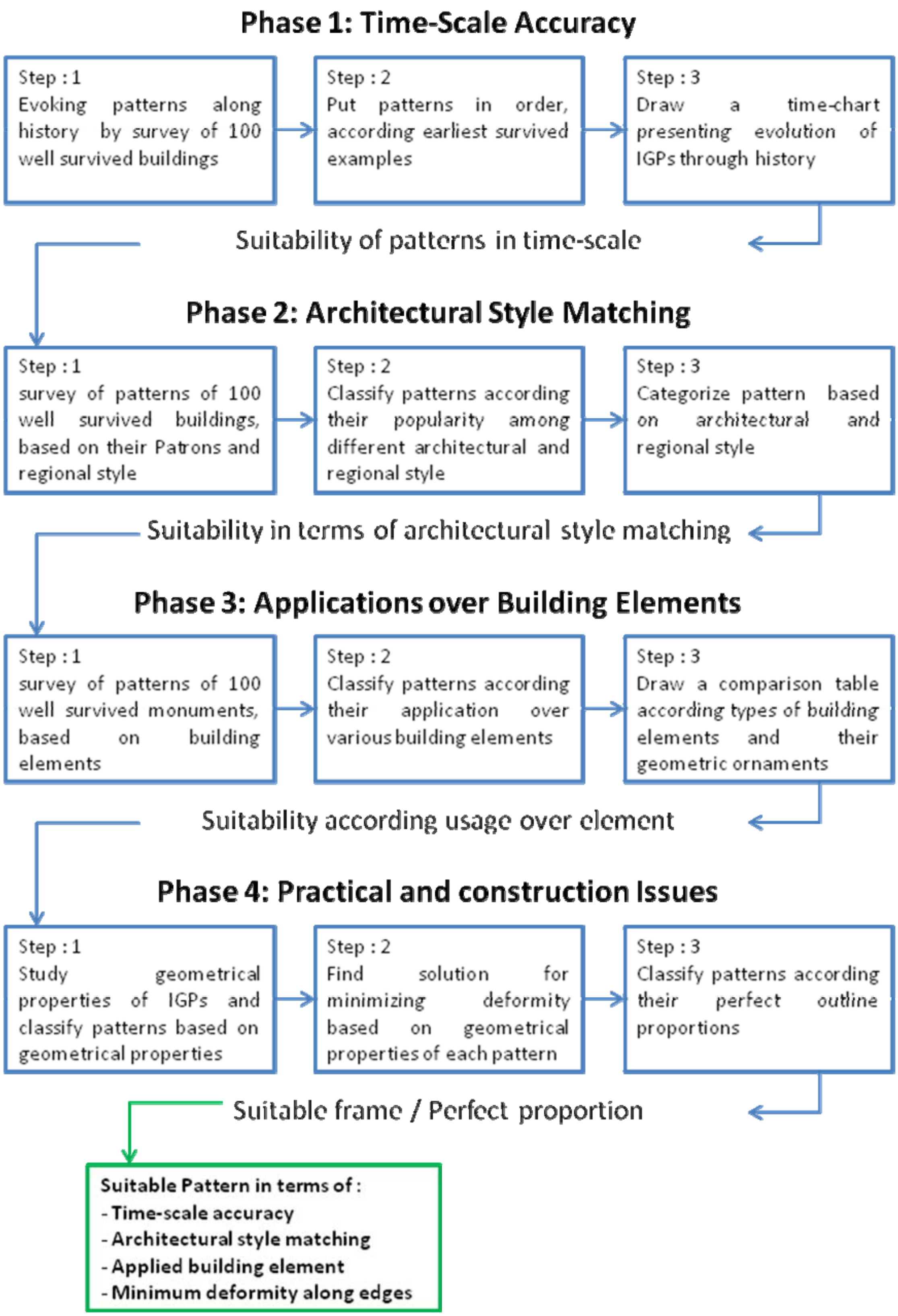
Table 1.2 Muslim Dynasties and Empires, which this study is focused on

	Style/Period	Capitals	Established	Disestablished
1	Umayyads	Damascus, Córdoba	660	750
2	Abbasid	Baghdad	750	1258
4	Fatimid	Mahdia, Cairo	909	1171
5	Seljuks	Ray, Isfahan, Hamadan, Kerman	1038	1194
6	Mamluk	Cairo	1250	1517
7	Ottoman	Söğüt, Bursa, Edirne, Constantinople	1290	1923
8	Safavid	Tabriz, Qazvin, Esfahan	1501	1736
9	Mughal	Agra; Fatehpur Sikri; Delhi	1526	1858

1.7.2. Analysis

Subsequently, application of patterns over above-mentioned buildings observed, recorded and documented according their date and region along with type of building elements (Such as dome, Minaret and etc.), which they are applied over. After collecting and documenting data, the **first analytical phase** is to tabulate patterns in terms of time-scale and put them in order. Eventually results of this phase lead us to draw a time-chart of evolution of patterns based on earliest survived examples. **Second analytical phase** was to discover and identify origin of patterns. In this stage, collected patterns are reclassified based on regionalism and where they are found. The result of this classification allows us to categorize patterns based on architectural and regional styles. **Third analytical phase** of this research is to characterize patterns according to where these patterns are mostly used in buildings. This will establish a guideline to use of IGPs upon building elements and would answer to “which parts of buildings are more decorated with patterns and what types of patterns are common for such elements?”. For this purpose, five main Islamic architectural elements have been selected. Dome would represent methods and techniques of decoration three-dimensional elements; Iwan is selected to cover two-dimensional elements such as walls and arched openings and windows. Mihrab is great example for Islamic interior designs and Minbar is a sample of Islamic building furniture. In addition, Minaret is selected to demonstrate accessibility and visual issues of decorating elements by geometrical patterns. As a result, third chapter of this research is focused on classifying and categorizing patterns by building elements, which they have been applied. While types of patterns in terms of style-period and place-to-apply are identified, it comes to question of “how to apply and adopt patterns for required surfaces?”. Hence, at **fourth analytical phase**, geometrical properties of patterns are analyzed based on geometrical issues that are related to architecture and ornaments. A detailed literature review on geometrical issues of architecture and ornaments in general and specifically IGPs is carried out in forth chapter of this research and as a result patterns ore classified according to their geometrical properties in fifth chapter. Moreover, in sixth chapter a guideline and method to simple way of recognizing suitability of patterns based on geometrical properties is provided.

Table 1.3 Analysis Workflow diagram of research process



1.8. Importance of Study

This research is an answer to ambiguities regarding both philosophical and practical aspects of suitability and adapting Islamic geometrical patterns. It traces evolution of Islamic patterns form early stages nearly modern era and identifies the origin of popular patterns. This research has collected the most prominent types of IGPs and categorized them according to their usage over building elements. Geometrical properties of patterns are also analyzed, and based on results, patterns on classified according to their geometrical properties. Eventually a guideline has been established to verify suitability of patterns in terms of style, period, types of elements that they are applied over and finally similarities and differences between patterns. Moreover, a method to adopt these patterns to required area and a quick way of recognizing type of patterns in term of geometrical properties is provided to help practical issues of using these types of ornaments.

1.9. Organization of Research

This report is organized into seven parts. Apart from first introductory and final conclusionary chapter, the five remaining chapters are sorted to address from the most fundamental issue of "when (evolution) and from where (origin)?" to last stage of "how to use?". In Second chapter, IGPs are studied historically based on both time-scale and period-styles, to draw time-chart of evolution of Islamic geometrical patterns. Third chapter is focused on building elements and categorizing patterns according to where they are mostly used in Islamic inspired buildings. In forth chapters, geometrical properties of patterns are studied, and as a result, in chapter five, patterns are classified according to their similarities and differences of geometrical properties. Throughout sixth chapter, a method for covering surfaces with minimum deformity along edges has been introduced. In addition, a simple way to find the perfect ratio for frame (outline boundary) of patterns is also introduced and illustrated with samples of patterns form different types of IGPs.

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